Generalization and prediction of Euler's Theorem for Homogeneous functions

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ABSTRACT

The paper deals with the prediction of higher order, homogeneous partial differential equations of Euler's theorem for two independent variables and generalization to Nth order. This theorem is also extended to 3rd order for m independent variables and a new formula is generated and verified. We write five rules for equation prediction called the Ajayous rules.

KEYWORDS

Homogenous functions, partial differential equation(PDE), order of PDE, degree of function, Euler's theorem, Ajayous rules, Equation prediction.

1.1 INTRODUCTION [1]

Leonhard Euler [5], a popular mathematician, physicist, astronomer, logician and engineer, lived between 1707 to 1783. He is regarded as "Analysis Incarnate". He is famous for his work known as Euler's Conjectures, Euler's formulae, Euler's identities, Euler's number, Euler's theorem. Though being a blind man in his later stages of his life he is regarded as one of the greatest mathematicians of all time. One of his greatest contributions is Euler's theorem for homogeneous functions.

Definition of a homogeneous function:

Let $u = u(x, y)$ be a homogeneous function, defined as

$$u(\lambda x, \lambda y) = \lambda^n u(x, y) \quad (1)$$

We will study examples for homogeneous and non-homogeneous functions.

Example 1.1:

$$u(x, y) = x^2 + y^2 \quad (2)$$

Now,

$$u(\lambda x, \lambda y) = (\lambda x)^2 + (\lambda y)^2 \quad (3)$$

$$u(\lambda x, \lambda y) = \lambda^2 (x^2 + y^2) \quad (4)$$

$$u(\lambda x, \lambda y) = \lambda^2 u(x, y) \quad (5)$$

Therefore, $u(x, y) = x^2 + y^2$ is a homogeneous function of degree 2.

Example 1.2:

$$u(x, y) = x^2 + y \quad (6)$$

Now,

$$u(\lambda x, \lambda y) = (\lambda x)^2 + \lambda y \quad (7)$$

$$u(\lambda x, \lambda y) = \lambda(\lambda x^2 + y) . \tag{8}$$

Here, $u(x, y) = x^2 + y$ cannot be expressed in the form $u(\lambda x + \lambda y) = \lambda^n u(x, y)$. Hence $u(x, y) = x^2 + y$ is not a homogeneous function.

Note on the symbols used:

In the following paper the most often used symbol will be of the form $u_{x^p y^q}$. This symbols mean that the homogeneous function u is partially differentiated with respect to x , ' p ' times and partially differentiated with respect to y , ' q ' times.

2. EULER'S THEOREM FOR HOMOGENEOUS FUNCTIONS OF TWO VARIABLES:

If $u = u(x, y)$ & $u(\lambda x, \lambda y) = \lambda^n u(x, y)$ then

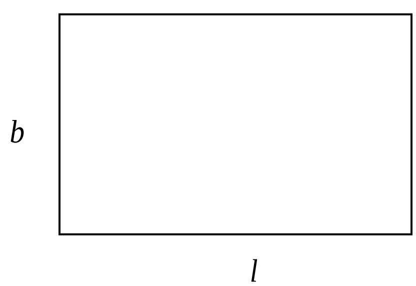
$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu \tag{9}$$

Importance of Euler's Theorem:

Euler's theorem builds a relationship between the dependent variable, the independent variables and their partial derivatives in an equation.

Verification of Euler's Theorem for area of rectangle:

Consider a rectangle with length ' l ' and breadth ' b '.



The area of the rectangle is $A = lb$

Also we can write $A = A(l, b)$.

Area of a rectangle being a function of length and breadth is an example for homogeneous function of degree '2'.

Using Euler's Theorem, we get

$$l\frac{\partial A}{\partial l} + b\frac{\partial A}{\partial b} = 2A \tag{10}$$

Substituting $A = lb$ in (9), we get

$$l\frac{\partial lb}{\partial l} + b\frac{\partial lb}{\partial b} = 2lb \tag{11}$$

$$lb\frac{\partial l}{\partial l} + bl\frac{\partial b}{\partial b} = 2lb \tag{12}$$

$$lb + bl = 2lb \tag{13}$$

$$2lb = 2lb \tag{14}$$

Hence, Euler's theorem is verified in this case.

2.1 Euler's Theorem for higher order partial differential equations:

According to Euler:

$$xu_x + yu_y = nu \tag{15}$$

Differentiate equation (9) partially with respect to x and then multiply it throughout by x,

$$x^2u_{x^2} + xu_x + xyu_{xy} = nxu_x \tag{16}$$

Differentiate equation (9) partially with respect to y and then multiply it throughout by y,

$$y^2u_{y^2} + yu_y + xyu_{xy} = nyu_y \tag{17}$$

Adding (16) & (17) and using (15), we get

$$x^2u_{x^2} + y^2u_{y^2} + 2xyu_{xy} = n(n-1)u \tag{18}$$

Equation (18) is the second order partial differential equation of Euler's theorem.

Here, from the 1st order partial differential equation the 2nd order partial differential equation is derived. In general , if we know the Euler's theorem for N th order then (N +1)th order partial differential equation of Euler's theorem can be derived following similar process as above.

Note: From now on the order of the partial differential equation be denoted as ' N '.

Continuing as above we can write Euler's theorem from N =1 to N =6.

$$xu_x + yu_y = nu \tag{19}$$

$$x^2u_{x^2} + y^2u_{y^2} + 2xyu_{xy} = n(n-1)u \tag{20}$$

$$x^3u_{x^3} + y^3u_{y^3} + 2x^2u_{x^2} + 2y^2u_{y^2} + 3x^2yu_{x^2y} + 3xy^2u_{xy^2} + 4xyu_{xy} = n^2(n-1)u \tag{21}$$

$$x^4u_{x^4}+y^4u_{y^4}+5x^3u_{x^3}+5y^3u_{y^3}+4x^2u_{x^2}+4y^2u_{y^2}+15xy^2u_{xy^2}+$$
$$15x^2yu_{x^2y}+4x^3yu_{x^3y}+4xy^3u_{xy^3}+6x^2y^2u_{x^2y^2}+8xyu_{xy}=n^3(n-1)u \quad (22)$$

$$x^5u_{x^5}+y^5u_{y^5}+9x^4u_{x^4}+9y^4u_{y4}+19x^3u_{x^3}+19y^3u_{y^3}+8x^2u_{x^2}+8y^2u_{y^2}+10x^2y^3u_{x^2y^3}+10x^3y^2u_{x^3y^2}+$$
$$36xy^3u_{xy^3}+36x^3yu_{x^3y}+57xy^2u_{xy^2}+57x^2yu_{x^2y}+5x^4yu_{x^4y}+5xy^4u_{xy^4}+54x^2y^2u_{x^2y^2}+16xyu_{xy}=n^4(n-1)u \quad (23)$$

$$x^6u_{x^6}+y^6u_{y^6}+14x^5u_{x^5}+14y^5u_{y^5}+55x^4u_{x^4}+55y^4u_{y^4}+65x^3u_{x^3}+65y^3u_{y^3}+$$
$$16x^2u_{x^2}+16y^2u_{y^2}+20x^3y^3u_{x^3y^3}+140x^2y^3u_{x^2y^3}+140x^3y^2u_{x^3y^2}+239xy^3u_{xy^3}+239x^3yu_{x^3y}+$$
$$15x^2y^4u_{x^2y^4}+15x^4y^2u_{x^4y^2}+70x^4yu_{x^4y}+70xy^4u_{xy^4}+6x^5yu_{x^5y}+6xy^5u_{xy^5}+195x^2yu_{x^2y}+195xy^2u_{xy^2}+$$
$$330x^2y^2u_{x^2y^2}+32xyu_{xy}=n^5(n-1)u \quad (24)$$

The interesting fact is that by careful observation of these six equations we observe that there exist patterns. By using these patterns, we can predict any partial differential equation of Euler's theorem for any 'N'. In the next section we write the five rules for the equation prediction called the Ajayous rules.

NOTE: Ajayous rules can be used only when $N \geq 2$.

2.2 General representation of Euler's Theorem:

The Ajayous rules for prediction of Partial differential equations of Euler's Theorem of N th order:

Ajayous rule 1:

The N th order partial differential equation of Euler's theorem is the form

$$\sum a_j x^p y^q u_{x^p y^q} = n^{N-1}(n-1)u \quad (25)$$

such that, $2 \leq p+q \leq N$, $0 \leq p \leq N$, $0 \leq q \leq N$

j varies from 1 to the total number of terms in the equation.

Also, $N \geq 2$. Where $p, q \in W$.

Ajayous rule 2:

The value of a_j (constant coefficients) corresponding to the terms $x^2u_{x^2}$ & $y^2u_{y^2}$ is determined using the formula:

$$2^{N-2} \quad (26)$$

Ajayous rule 3:

The value of a_j (constant coefficients) corresponding to the term xyu_{xy} is determined using the formula:

$$2^{N-1} \quad (27)$$

Ajayous rule 4:

The value of a_j (constant coefficients) corresponding to the terms $x^3u_{x^3}$ & $y^3u_{y^3}$ is determined using the formula:

$$1-\frac{31}{4}(N-3)+\frac{181}{12}(N-3)^2-\frac{47}{6}(N-3)^3+\frac{23}{12}(N-3)^4 \tag{28}$$

Ajayous rule 5: The remaining values of a_j corresponding other few terms are determined by comparing the left hand side of the partial differential equation to the expansion of $(x+y)^N$ and equating with similar terms.

2.3 Illustrations of Ajayous rules:

The above mentioned 5 rules are illustrated by using the following examples.

Example 2.1:

Let us write Euler's theorem using Ajayous rules when N =2.

Using Ajayous rule 1:

Here,

$$2 \le p+q \le 2 \quad \text{or} \quad p+q=2$$

$$0 \le p \le 2$$
$$0 \le q \le 2$$

So the Euler's equation will be of the form,

$$a_1x^2u_{x^2}+a_2y^2u_{y^2}+a_3xyu_{xy}=n(n-1)u \tag{29}$$

Using Ajayous rule 2:

The values of a_1 & a_2 which are constant coefficients of $x^2u_{x^2}$ & $y^2u_{y^2}$ will be 2^{2-2} =1 ($\because$ N =2).

Using Ajayous rule3:

The value of a_3 which is the constant coefficient of xyu_{xy} will 2^{2-1} =2

Since we ended up with all the terms we can skip Ajayous rule 4 &5.

Substituting the values of a_1, a_2 & a_3 in (29), we get

$$x^2u_{x^2}+y^2u_{y^2}+2xyu_{xy}=n(n-1)u \tag{30}$$

Hence, we have predicted an PDE of Euler's theorem without deriving but by using Ajayous rules.

Let us verify this equation by substituting a homogeneous function.

Suppose

$u = x^2 + y^2$

Then

LHS= $x^2(2) + y^2(2) + 2xy(0) = 2(x^2 + y^2)$

RHS=2(2-1)u= $2(x^2 + y^2)$.

Hence, LHS=RHS.

EXAMPLE 2.2:

Let us write the Euler's theorem using Ajayous rules when N =3.

Using Ajayous rule 1:

Here,

$$0 \le p \le 3, 0 \le q \le 3,$$

$$2 \le p + q \le 3.$$

So the Euler's equation will be of the form:

$$a_1 x^3 u_{x^3} + a_2 y^3 u_{y^3} + a_3 x^2 u_{x^2} + a_4 y^2 u_{y^2} + a_5 x^2 y u_{x^2 y} + a_6 x y^2 u_{xy^2} + a_7 xy u_{xy} = n^2 (n-1) u . \quad (31)$$

Using Ajayous rule 2:

The values of a_3 & a_4 ,which happen to be the constant coefficients of $x^2 u_{x^2}$ & $y^2 u_{y^2}$ are 2^{3-2}=2.

Using Ajayous rule 3:

The value of a_7,which is the constant coefficient of xyu_{xy} is 2^{3-1}=4.

Using Ajayous rule 4:

The values of a_1 & a_2, the constant coefficients of $x^3 u_{x^3}$ & $y^3 u_{y^3}$ are

$1 - \frac{31}{4}(3-3) + \frac{181}{12}(3-3)^2 - \frac{47}{6}(3-3)^3 + \frac{23}{12}(3-3)^4$=1.

Using Ajayous rule 5:

We have to write the expansion of $(x+y)^3$.

$$(x+y)^3 = x^3 + y^3 + 3x^2 y + 3xy^2 . \quad (32)$$

Comparing (32) with the (31) and equating coefficients of similar terms, we find that the values of a_5&a_6 are 3.

$3x^2 y$ in the binomial expansion is compared with $a_5 x^2 y u_{x^2 y}$.

Similarly, $3xy^2$ is compared with $a_6 xy^2 u_{xy^2}$.

Hence, the values of a_5 & a_6 are 3.

Substituting the values found out using the Ajayous rules in (31), we get

$$x^3u_{x^3} + y^3u_{y^3} + 2x^2u_{x^2} + 2y^2u_{y^2} + 3x^2yu_{x^2y} + 3xy^2u_{xy^2} + 4xyu_{xy} = n^2(n-1)u \tag{33}$$

Let us verify equation (33) by substituting a homogeneous function.

Suppose $u = x^2y^2$

Then, we get

LHS=$48x^2y^2$

RHS=$48x^2y^2$

Therefore, LHS=RHS.

EXAMPLE 2.3:

Let us write the PDE of Euler's theorem when N =4.

Using Ajayous rule 1:

Here,

$$0 \le p \le 4,$$
$$0 \le q \le 4,$$
$$2 \le p + q \le 4,$$

Therefore, the Euler's theorem for N =4 will be in the form

$$\begin{aligned} &a_1x^4u_{x^4} + a_2y^4u_{y^4} + a_3x^3u_{x^3} + a_4y^3u_{y^3} + a_5x^2u_{x^2} + a_6y^2u_{y^2} + a_7xy^2u_{xy^2} + \\ &a_8x^2yu_{x^2y} + a_9x^3yu_{x^3y} + a_{10}xy^3u_{xy^3} + a_{11}x^2y^2u_{x^2y^2} + a_{12}xyu_{xy} = n^3(n-1)u \end{aligned} \tag{34}$$

Using Ajayous rule 2:

The values of a_5 & a_6 will be 2^{4-2} =4

Using Ajayous rule 3:

The value of a_{12} will be 2^{4-1}=8

Using Ajayous rule 4:

The values of a_3 & a_4 will be $1 - \frac{31}{4}(4-3) + \frac{181}{12}(4-3)^2 - \frac{47}{6}(4-3)^3 + \frac{23}{12}(4-3)^4$=5.

Using Ajayous rule 5:

We need to write the expansion of $(x+y)^4$,

$$(x+y)^4 = x^4 + y^4 + 4x^3y + 4xy^3 + 6x^2y^2.$$

By comparing the PDE of Euler's theorem with the above expansion we find that

$$a_1 = 1,$$
$$a_2 = 1,$$
$$a_9 = 4,$$
$$a_{10} = 4,$$
$$a_{11} = 6$$

Substituting the values found out from Ajayous rules in (34) ,we get

$$\begin{aligned} &x^4u_{x^4} + y^4u_{y^4} + 5x^3u_{x^3} + 5y^3u_{y^3} + 4x^2u_{x^2} + 4y^2u_{y^2} + a_7xy^2u_{xy^2} + \\ &a_8x^2yu_{x^2y} + 4x^3yu_{x^3y} + 4xy^3u_{xy^3} + 6x^2y^2u_{x^2y^2} + 8xyu_{xy} = n^3(n-1)u \end{aligned} \tag{35}$$

Clearly, from Ajayous rules the values of a_7 & a_8 are not determined. That is the limitation.

Assuming the values for a_7 & a_8 as 15, let us verify (35) by substituting a homogeneous function.

$$u = x^4$$
$$u_x = 4x^3$$
$$u_{x^2} = 12x^2$$
$$u_{x^3} = 24x$$
$$u_{x^4} = 24$$

Substituting these values in (35) will yield,

LHS= $192x^4$

RHS=

$$4^3(4-1)x^4$$
$$= 192x^4$$

Hence LHS=RHS.

EXAMPLE 2.4:

Let us write the PDE of Euler's theorem when N =5.

Using Ajayous rule 1:

Here,

$$0 \le p \le 5,$$
$$0 \le q \le 5,$$
$$2 \le p+q \le 5$$

Therefore, the Euler's theorem will be of the form:

$$a_1 x^5 u_{x^5} + a_2 y^5 u_{y^5} + a_3 x^4 u_{x^4} + a_4 y^4 u_{y4} + a_5 x^3 u_{x^3} + a_6 y^3 u_{y^3} + a_7 x^2 u_{x^2} + a_8 y^2 u_{y^2} + a_9 x^2 y^3 u_{x^2 y^3} + a_{10} x^3 y^2 u_{x^3 y^2} + a_{11} xy^3 u_{xy^3} + a_{12} x^3 y u_{x^3 y} + a_{13} xy^2 u_{xy^2} + a_{14} x^2 y u_{x^2 y} + a_{15} x^4 y u_{x^4 y} + a_{16} xy^4 u_{xy^4} + a_{17} x^2 y^2 u_{x^2 y^2} + a_{18} xyu_{xy} = n^4 (n-1) u \quad (36)$$

Using Ajayous rule 2:

The values of $a_7 \,\&\, a_8$ are $2^{5-2} = 8$.

Using Ajayous rule 3:

The value of $a_{18} = 2^{5-1} = 16$.

Using Ajayous rule 4:

The values of $a_5 \,\&\, a_6$ are $1 - \frac{31}{4}(5-3) + \frac{181}{12}(5-3)^2 - \frac{47}{6}(5-3)^3 + \frac{23}{12}(5-3)^4 = 19$.

Using Ajayous rule 5:

We need to write the expansion of $(x+y)^5$,

$$(x+y)^5 = x^5 + y^5 + 10x^2 y^3 + 10x^3 y^2 + 5x^4 y + 5xy^4 \quad (37)$$

By comparing (36) with (37) and equating the coefficients of similar terms, we get

$$a_1 = 1,$$
$$a_2 = 1,$$
$$a_9 = 10,$$
$$a_{10} = 10,$$
$$a_{15} = 5,$$
$$a_{16} = 5$$

Substituting all the values of constant coefficients determined by Ajayous rules in (36), we get

$$x^5 u_{x^5} + y^5 u_{y^5} + a_3 x^4 u_{x^4} + a_4 y^4 u_{y4} + 19x^3 u_{x^3} + 19y^3 u_{y^3} + 8x^2 u_{x^2} + 8y^2 u_{y^2} + 10x^2 y^3 u_{x^2 y^3} + 10x^3 y^2 u_{x^3 y^2} + a_{11} xy^3 u_{xy^3} + a_{12} x^3 y u_{x^3 y} + a_{13} xy^2 u_{xy^2} + a_{14} x^2 y u_{x^2 y} + 5x^4 y u_{x^4 y} + 5xy^4 u_{xy^4} + a_{17} x^2 y^2 u_{x^2 y^2} + 16xyu_{xy} = n^4 (n-1) u \quad (38)$$

Clearly, the values of $a_{11}, a_{12}, a_{13}, a_{14}, a_{17}$ remain undetermined by Ajayous rules.

LIMITATIONS OF AJAYOUS RULES:

- Ajayous rules can be used to write down the form of an PDE of Euler's theorem only when $N \geq 2$.
- Ajayous rules can be used to determine the constant coefficients of the PDE only for the terms $x^2 u_{x^2} / y^2 u_{y^2}, xyu_{xy}, x^3 u_{x^3} / y^3 u_{y^3}$ and those terms of the form $x^p y^q u_{x^p y^q}$ such that $p+q=N$.

3. EULER'S THEOREM FOR HOMOGENEOUS FUNCTION OF 'm' VARIABLES:

If $xu_x + yu_y = nu$ represents Euler's theorem for two variables *x* & *y* then, Euler's theorem for '*m*'variables would be

$$x_1 u_{x_1} + x_2 u_{x_2} + x_3 u_{x_3} + \dots\dots + x_m u_{x_m} = nu \tag{39}$$

where $u = u(x_1, x_2, x_3, \dots\dots, x_m)$. (40)

Similarly, the Euler's theorem for N=2 having 'm' variables will be

$$\begin{aligned} & {x_1}^2 u_{x_1^2} + {x_2}^2 u_{x_2^2} + {x_3}^2 u_{x_3^2} + \dots\dots + {x_m}^2 u_{x_m^2} + 2x_1x_2u_{x_1x_2} + 2x_1x_3u_{x_1x_3} + 2x_1x_4u_{x_1x_4} + \dots\dots + 2x_1x_mu_{x_1x_m} + \\ & 2x_2x_3u_{x_2x_3} + 2x_2x_4u_{x_2x_4} + 2x_2x_5u_{x_2x_5} + \dots\dots + 2x_2x_mu_{x_2x_m} + 2x_3x_4u_{x_3x_4} + 2x_3x_5u_{x_3x_5} + 2x_3x_6u_{x_3x_6} + \dots\dots\dots 2x_3x_mu_{x_3x_m} + \\ & \dots\dots\dots + 2x_{m-1}x_mu_{x_{m-1}x_m} = n(n-1)u \end{aligned} \tag{41}$$

simplifying the above equation by using summation notation, we write

$$\sum_{i=1}^{m} {x_i}^2 u_{x_i^2} + 2x_1 \sum_{i=2}^{m} x_i u_{x_1x_i} + 2x_2 \sum_{i=3}^{m} x_i u_{x_2x_i} + 2x_3 \sum_{i=4}^{m} x_i u_{x_3x_i} + \dots\dots + 2x_{m-1}x_m u_{x_{m-1}x_m} = n(n-1)u \tag{42}$$

The equation (42) can be used when $m \geq 2$. But while verifying the PDE a homogeneous function of less than two variables can be substituted.

Now, let us arrive at the PDE of Euler's theorem for N =2 when number of variables *m*=2:

Substituting m=2 in equation (42), we get

$$\sum_{i=1}^{2} {x_i}^2 u_{x_i^2} + 2x_1 \sum_{i=2}^{2} x_i u_{x_1x_i} = n(n-1)u \tag{43}$$

$${x_1}^2 u_{x_1^2} + {x_2}^2 u_{x_2^2} + 2x_1x_2u_{x_1x_2} = n(n-1)u \tag{44}$$

This is the Euler's theorem for N=2 of two variable functions which we are very well familiar.

When m=3, we get

$$\sum_{i=1}^{3} {x_i}^2 u_{x_i^2} + 2x_1 \sum_{i=2}^{3} x_i u_{x_1x_i} + 2x_2 \sum_{i=2}^{3} x_i u_{x_2x_i} = n(n-1)u \tag{45}$$

$${x_1}^2 u_{x_1^2} + {x_2}^2 u_{x_2^2} + {x_3}^2 u_{x_3^2} + 2x_1x_2u_{x_1x_2} + 2x_1x_3u_{x_1x_3} + 2x_2x_3u_{x_2x_3} = n(n-1)u \tag{46}$$

Let us verify equation (46) by substituting a homogeneous function, Let u= $x_1 + x_2 + x_3$ then

$$x_1^{\,2}(0) + x_2^{\,2}(0) + x_3^{\,2}(0) + 2x_1x_2(0) + 2x_1x_3(0) + 2x_2x_3(0) = 1(1-1)(x_1 + x_2 + x_3)$$

0=0.

Hence, equation (46) is verified.

When m=4, we get

$$\sum_{i=1}^{4} x_i^{\,2} u_{x_i^2} + 2x_1 \sum_{i=2}^{4} x_i u_{x_1x_i} + 2x_2 \sum_{i=2}^{4} x_i u_{x_2x_i}\; 2x_3 \sum_{i=2}^{4} x_i u_{x_3x_i} = n(n-1)u \qquad (47)$$

$$x_1^{\,2}u_{x_1^2} + x_2^{\,2}u_{x_2^2} + x_3^{\,2}u_{x_3^2} + x_4^{\,2}u_{x_4^2} + 2x_1x_2u_{x_1x_2} + 2x_1x_3u_{x_1x_3} + 2x_2x_3u_{x_2x_3} + 2x_2x_4u_{x_2x_4} + 2x_3x_4u_{x_3x_4} = n(n-1)u \qquad (48)$$

let us verify equation (48) by substituting a homogenous function,

Suppose *u*= $x_1^{\,2} + x_2^{\,2} + x_3^{\,2} + x_4^{\,2}$

then, we get

$$2(x_1^{\,2} + x_2^{\,2} + x_3^{\,2} + x_4^{\,2}) = 2(x_1^{\,2} + x_2^{\,2} + x_3^{\,2} + x_4^{\,2}).$$

Therefore, LHS=RHS.

The Euler's theorem for homogenous functions of '*m*' variables when N =3 is

$$\sum_{i=1}^{m} x_i^{\,3} u_{x_i^3} + 2\sum_{i=1}^{m} x_i^{\,2} u_{x_i^2} + 3x_1^{\,2} \sum_{i=2}^{m} x_i u_{x_1^{\,2}x_i} + 3x_2^{\,2} \sum_{i=1}^{m} x_i u_{x_2^{\,2}x_i} + 3x_3^{\,2} \sum_{i=1}^{m} x_i u_{x_3^{\,2}x_i} + \ldots\ldots\ldots +$$
$$3x_m^{\,2} x_{m-1} u_{x_m^2 x_{m-1}} + 4x_1 \sum_{i=2}^{m} x_i u_{x_1x_i} + 4x_2 \sum_{i=3}^{m} x_i u_{x_2x_i} + 4x_3 \sum_{i=4}^{m} x_i u_{x_3x_i} + \ldots\ldots + 4x_m x_{m-1} u_{x_m x_{m-1}}$$
$$+6x_1x_2 \sum_{i=3}^{m} x_i u_{x_1x_2x_i} + 6x_1x_3 \sum_{i=4}^{m} x_i u_{x_1x_3x_i} + 6x_1x_4 \sum_{i=5}^{m} x_i u_{x_1x_4x_i} + \ldots\ldots\ldots\ldots + 6x_1 x_m x_{m-1} u_{x_1 x_m x_{m-1}} \qquad (49)$$
$$+6x_2x_3 \sum_{i=4}^{m} x_i u_{x_2x_3x_i} + 6x_2x_4 \sum_{i=5}^{m} x_i u_{x_2x_4x_i} + 6x_2x_5 \sum_{i=6}^{m} x_i u_{x_2x_5x_i} + \ldots\ldots\ldots\ldots + 6x_2 x_m x_{m-1} u_{x_2 x_m x_{m-1}} +$$
$$\ldots\ldots\ldots\ldots + 6x_m x_{m-1} x_{m-2} u_{x_m x_{m-1} x_{m-2}} = n^2(n-1)u$$

In equation (49) summation is carried out such that' no' term is of the form $x_p^{\,2} x_q u_{x_p^2 x_q}$ such that *p*=*q*.

Let us verify equation (49) with examples.

Example 3. 1:

When m=3, then equation (49) becomes,

$$\sum_{i=1}^{3} x_i^3 u_{x_i^3} + 2\sum_{i=1}^{3} x_i^2 u_{x_i^2} + 3x_1^2 \sum_{i=2}^{3} x_i u_{x_1^2 x_i} + 3x_2^2 \sum_{i=1}^{3} x_i u_{x_2^2 x_i} + 3x_3^2 \sum_{i=1}^{3} x_i u_{x_3^2 x_i}$$
$$+4x_1 \sum_{i=2}^{3} x_i u_{x_1 x_i} + 4x_2 \sum_{i=3}^{3} x_i u_{x_2 x_i} + 6x_1 x_2 \sum_{i=3}^{3} x_i u_{x_1 x_2 x_i} = n^2(n-1)u \qquad (50)$$

Simplifying the equation (50), we get

$$x_1^3 u_{x_1^3} + x_2^3 u_{x_2^3} + x_3^3 u_{x_3^3} + 2x_1^2 u_{x_1^2} + 2x_2^2 u_{x_2^2} + 2x_3^2 u_{x_3^2} + 3x_1^2 x_2 u_{x_1^2 x_2}$$
$$+3x_1^2 x_3 u_{x_1^2 x_3} + 3x_2^2 x_1 u_{x_2^2 x_1} + 3x_2^2 x_3 u_{x_2^2 x_3} + 3x_3^2 x_1 u_{x_3^2 x_1} + 3x_3^2 x_2 u_{x_3^2 x_2} \qquad (51)$$
$$+4x_1 x_2 u_{x_1 x_2} + 4x_1 x_3 u_{x_1 x_3} + 4x_2 x_3 u_{x_2 x_3} + 4x_2 x_4 u_{x_2 x_4} + 6x_1 x_2 x_3 u_{x_1 x_2 x_3} = n^2(n-1)u$$

Let us substitute a homogeneous function to verify equation (51).

Suppose

u= $x_1^3 + x_2^3 + x_3^3$

then

LHS=$18(x_1^3 + x_2^3 + x_3^3)$

And RHS=$18(x_1^3 + x_2^3 + x_3^3)$.

4. CONCLUSION

All that could be done in this paper is that to predict PDEs of Euler's theorem using what is named the Ajayous rules. Also to generalize the 2nd and 3rd order PDEs of Euler's theorem to 'm' variables.

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